Int. J. Heat Mass Transfer. Vol. 17, pp. 1349–1356. Pergamon Press 1974. Printed in Great Britain

EFFECTS OF DENSITY INVERSION ON FREE CONVECTIVE HEAT TRANSFER IN POROUS LAYER HEATED FROM BELOW

YIN-CHAO YEN*
U.S. Army Cold Regions Research and Engineering Laboratory,
Hanover, New Hampshire 03755, U.S.A.

(*Received* 21 *September* 1973 *and in revised form* 31 *January* 1974)

Abstract—A study was conducted to investigate the effects of density inversion on free convective heat transfer in a porous layer heated from below. Glass beads in water composed the porous medium. For upper boundary at 4 and 8°C, thus eliminating the effect of density inversion on the onset of convection, the critical Rayleigh number was found to be $4\pi^2$. The effect of density inversion was evaluated by maintaining the upper boundary temperature at 0°C. The onset of convection was found to be dependent on two thermal parameters which are functions of the boundary temperatures and the coefficients representing the fluid density–temperature relation. The Nusselt number can be represented in terms of a modified Rayleigh number. The effect of density inversion on heat-transfer rate was found to be significant and to decrease as the temperature difference across the layer increases. For small ΔT, the effect of density inversion causes heat-transfer rate to be as much as 100 per cent less than under non-density inversion conditions.

NOMENCLATURE

A, parameter defined as $(T_l - T_m)/(T_l - T_u)$ [dimensionless];
c_p, specific heat [J/kg . K];
d, glass bead diameter [mm];
D, layer depth [mm];
ΔT, temperature difference across the porous layer [degC];
g, gravitational constant [m/s²];
k_m, thermal conductivity of the fluid and solid matrix [W/m . K];
k_f, thermal conductivity of the fluid [W/m . K];
k_s, thermal conductivity of the solid [W/m . K];
K, permeability defined in equation (3) [m²];
Nu, Nusselt number, $= hD/k_m$ [dimensionless];
Ra, Rayleigh number, $= g\gamma\Delta T D^3/\kappa\alpha_m$ [dimensionless];
R_c, critical Rayleigh number for $R = 4\pi^2$ [dimensionless];
Ra_m, modified Rayleigh number defined in equation (2) [dimensionless];
$(Ra_m)_c$, modified critical Rayleigh number [dimensionless];
T, temperature [degC];
T_m, temperature at maximum density [4 degC];
T_l, lower boundary temperature [degC];
T_u, upper boundary temperature [degC].

Greek symbols

β, temperature gradient [degC/m];
ε, porosity (ratio of void volume to total volume) [dimensionless];
ρ, fluid density [kg/m³];
ρ_m, fluid density at T_m [kg/m³];
γ, coefficient of volumetric expansion [1/degC];
γ_1, coefficient in equation (1) [1/degC²];
γ_2, coefficient in equation (1) [1/degC³];
α_m, thermal diffusivity of the fluid and solid matrix, $k_m/(\rho c_p)_f$ [m²/s];
λ_1, parameter in equation (4) [dimensionless];
λ_2, parameter in equation (5) [dimensionless];
κ, kinematic viscosity [m²/s].

Subscripts

f, fluid;
c, critical;
m, matrix, modified;
s, solid;
l, lower;
u, upper;
0, at 0 degC;
4, at 4 degC;
8, at 8 degC.

*Also Visiting Professor of Chemical Engineering at the University of New Hampshire, Durham, New Hampshire 03824, U.S.A.

1. INTRODUCTION

THE PROBLEM of free convective heat transfer and fluid flow has been the subject of extensive study. Early theoretical and experimental studies of convective flow of a fluid in a porous medium were made by Horton and Rogers [1]; Morrison *et al.* [2]; Rogers and Morrison [3]; and Rogers *et al.* [4] during the period 1945–1951 in connection with NaCl distribution in subterranean sand layers. Lapwood [5] in 1948 solved a similar problem independently, in a more precise manner, and established the criterion for the onset of convection, which was confirmed experimentally by Katto and Masuoka [6]. Numerical investigation of the two-dimensional problem by Elder [7] and Karra [8] also verified the critical Rayleigh number $4\pi^2$ obtained by Lapwood. In these studies, the working fluid has a linear density-temperature relationship.

Stability analyses of horizontal liquid layers with a density maximum were made by Debler [9], Vernois [10], and Tien [11] using a parabolic density–temperature relationship. A more recent study was performed by Sun *et al.* [12] using a cubic density–temperature relationship. The work of Sun *et al.*, was extended recently to include the effect of density maximum on the onset of convection in a porous medium [13]. In their work, a linear stability analysis was made, and the empirical expression of Darcy's law was used to describe the fluid flow for liquids possessing a density–temperature relationship

$$\rho = \rho_m[1-\gamma_1(T-T_m)^2-\gamma_2(T-T_m)^3]. \quad (1)$$

A modified Rayleigh number was derived as

$$Ra_m = \frac{2D^2 g\beta\gamma_1 A(\Delta T)\left[1+\left(\frac{3}{2}\frac{\gamma_2}{\gamma_1}\right)A(\Delta T)\right]K}{\kappa\alpha_m} \quad (2)$$

where K, according to Black–Kozeny, is given as

$$K = \frac{\varepsilon^3 d^2}{150(1-\varepsilon)^2}. \quad (3)$$

The onset of convection was found to be dependent on two parameters, λ_1 and λ_2, expressed respectively as

$$\lambda_1 = -\left(\frac{1}{A}\right)\frac{\left[1+3\left(\frac{\gamma_2}{\gamma_1}\right)A(\Delta T)\right]}{\left[1+\frac{3}{2}\left(\frac{\gamma_2}{\gamma_1}\right)A(\Delta T)\right]} \quad (4)$$

and

$$\lambda_2 = \left(\frac{1}{A^2}\right)\frac{\left(\frac{3}{2}\frac{\gamma_2}{\gamma_1}\right)A(\Delta T)}{\left[1+\left(\frac{3}{2}\frac{\gamma_2}{\gamma_1}\right)A(\Delta T)\right]}. \quad (5)$$

These two parameters can be determined from T_l, T_u, γ_1 and γ_2.

Masuoka [14] recently reported a theoretical and experimental study on heat transfer by free convection in a porous layer heated from below. The porous medium was composed of water and glass beads (1·85 and 3·12 mm in mean diameter) with the layer depth ranging from 30 to 100 mm. He concluded that the heat transfer near the critical state of the occurrence of convection can be expressed by

$$Nu = 1+2\left[1-\frac{4\pi^2}{R}\right] \quad (6)$$

where $R = Ra\,.\,K/D^2$, and Ra is the classical Rayleigh number defined as $g\gamma(T_l-T_u)D^3/\kappa\alpha_m$. The above expression compared well with his experimental results for R up to $2R_c (R_c = 4\pi^2)$. When R exceeds from $2R_c$ to $3R_c$, Nu values become independent of both the layer depth and the thermal conductivity of the porous medium and become directly proportional to the parameter R.

Literature surveys have failed to reveal any prior experimental work on heat transfer of fluids exhibiting a density inversion in a porous layer. The purpose of this investigation is twofold: (1) To confirm the analytical predictions that the classical Rayleigh number, defining the onset of convection of normal fluids in a porous layer, must be modified with a dependency upon the parameters λ_1 and λ_2 to account for the effects of fluid density maximum on convection. These parameters were found to be functions of the thermal boundary conditions and the coefficient representing the density–temperature relation of the fluid. (2) To evaluate the effect of density inversion on the rate of heat transfer in the porous layer by maintaining T_u at 0 degC and >4 degC respectively. This research is motivated by the interest in determining the effect of free convection on heat transfer within permafrost during freezing and thawing.

2. EXPERIMENTAL APPARATUS AND PROCEDURE

The experimental apparatus used in this work is essentially the same as the one described in detail by Tien *et al.* [15].

Glass beads of two mean diameters (3·0 and 6·0 mm) were used. The beads were washed thoroughly with organic solvent and rinsed several times with distilled water. After drying completely, the beads were allowed to fill the porous layer chamber by free fall. The weight of the beads was recorded, and by knowing the total layer volume and the bulk density of the beads, the porosity of the layer was calculated. Table 1 summarizes the porosity and its variation with the ratio of bead diameter d to layer depth D.

Table 1. Variation of ε with D and d

D(mm)	$d = 3{\cdot}0$ mm		$d = 6{\cdot}0$ mm	
	ε	d/D	ε	d/D
6·35	0·440	0·473		0·946
25·40	0·388	0·118	0·398	0·236
50·80	0·376	0·058	0·391	0·116
76·20	0·374	0·040		0·080

The porosity ε, the ratio of the pore space to the whole volume of the porous medium, has a tendency to increase with d/D, as can be seen from the table. This is due to the pore space effect near the lower and upper boundary surfaces which is about 20 per cent greater than that within the porous layer. However, the local difference of the pore space is not great, so that its effect on the mean porosity is substantially negligible if d/D is less than 0·20.

After the glass beads were packed into the porous layer chamber, the apparatus was assembled and levelled. It was tilted about 45° so that one of the brass tubes was at the highest point in the chamber. Then the porous layer was filled by connecting one tube to a vacuum pump and the other to a flask of distilled water which was boiled for a few hours to make it air-free. The water was drawn slowly through a heat exchanger consisting of coiled copper tubing immersed in a water–ice bath before it entered the chamber. This step was considered to be essential in eliminating vapor condensation on the chamber walls. As soon as the layer was filled up completely, the vacuum pump stopped. Before initiation of a series of experimental runs, care was taken to insure that no air bubbles were within the porous layer.

The glycol flow was initiated and the power was set at various levels from 10 to 150 W. The glycol bath temperature was adjusted so that the upper copper plate could be maintained at 0, 4 and 8 degC. All temperatures were continuously recorded. Six to eight hours were usually required to reach the steady state.

3. EXPERIMENTAL RESULTS AND DISCUSSION

In all, 158 experimental runs were conducted for various layer depths; of these, 74 were carried out by maintaining the upper boundary temperature at 4 and 8 degC. The difference between the two thermocouple readings of the heated plate was found to be dependent on the power input to the system. The maximum variation in temperature occurred when the power input was the highest. But in no case did the variation exceed 1 degC in the warm plate. The arithmetic mean of the two temperature readings of the plate was used to evaluate the temperature differential across the layer. The accuracy of the temperature reading was $\pm 0{\cdot}1$ degC. From the temperature readings of the thermocouples epoxied to the inside and outside of the bottom and side Lucite walls, the heat loss under steady-state conditions was evaluated.

The heat loss was found to be dependent on the layer depth and the bead diameter. Greater losses were observed for greater depths and larger filling particle diameter, reaching a maximum loss of about 4 per cent. To insure that the technique used to account for the heat loss was adequate, a few experimental runs were conducted with room temperature adjusted to the heated plate temperature, thus eliminating the heat loss. For identical power inputs, Nusselt numbers were computed and found to be equal to those under heat loss conditions, indicating the adequacy of the heat loss estimation.

The thermal conductivity values, k_m used in evaluating the Nusselt number, parameter $R = RaK/D^2$, and modified Rayleigh number Ra_m are evaluated from the expression derived by Kunii and Smith [16]

$$\frac{k_m}{k_f} = \left[\varepsilon + \frac{0{\cdot}90(1-\varepsilon)}{0{\cdot}32 + \dfrac{2}{3}\dfrac{k_f}{k_s}} \right]. \tag{7}$$

As indicated in Table 1, the porosity is near constant for all layer depths under investigation, (except for $D = 6{\cdot}35$ mm) and can be taken as 0·385. The value of k_s is taken to be 0·81 W/m.K. The value of k_f and all other physical properties are evaluated at the arithmetic mean temperatures of T_l and T_u. The Blake–Kozeny equation is used to describe the permeability of the medium. The temperature differences across the porous layers ranged from 6 to 56 degC. Figure 1 shows a plot of Nu vs R for all the experiments with T_u maintained at 4 and 8 degC. When the temperature difference is relatively small, the liquid is in stable equilibrium due to its viscosity, i.e. the fluid is at rest and the heat is transferred by conduction. One can observe that in the vicinity where $R = Ra.K/D^2$ exceeds the critical value of $4\pi^2$, developed theoretically by Lapwood [5] and confirmed experimentally by Katto and Masuoka [6], convection occurs.

As predicted and confirmed by Masuoka [14], the heat transfer above the critical state seems to be divided into two regions. Since few experiments have been conducted in the transition region, ($4\pi^2 < R < 100$), there can be no conclusive verification of Masuoka's findings. For $R > 100$, Nu is found to be proportional to R, and can be represented by $Nu = 0{\cdot}05R^{0{\cdot}82}$ with a correlation coefficient of 0·983. Previous results by Elder [7], Masuoka [14], Schneider [17], and Gupta and Joseph [18] are shown in Fig. 2 along with the results of this study. The experimental results of Combrarnous and LeFur [19] and Buretta and Berman

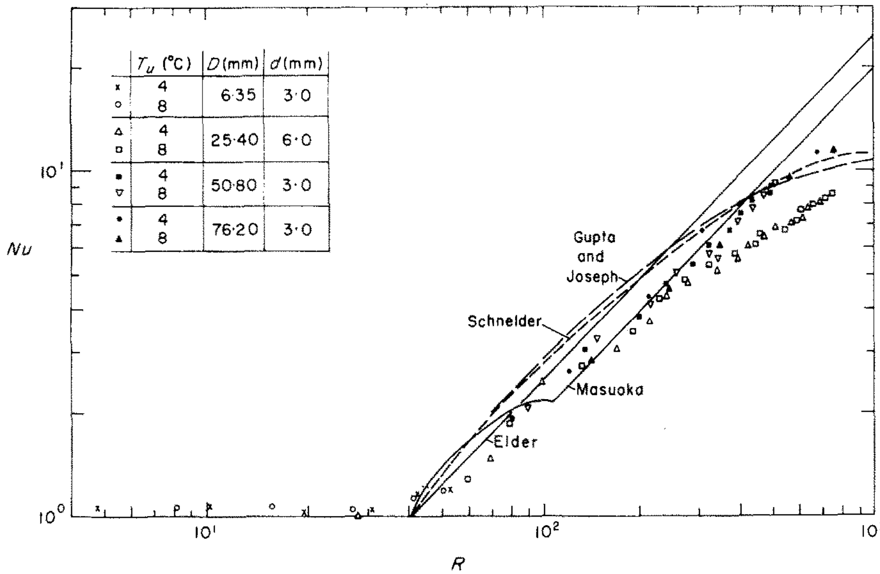

FIG. 1. Relationship between Nu and R for $T_u = 4$ and 8 degC, and comparison of results with those of previous investigators.

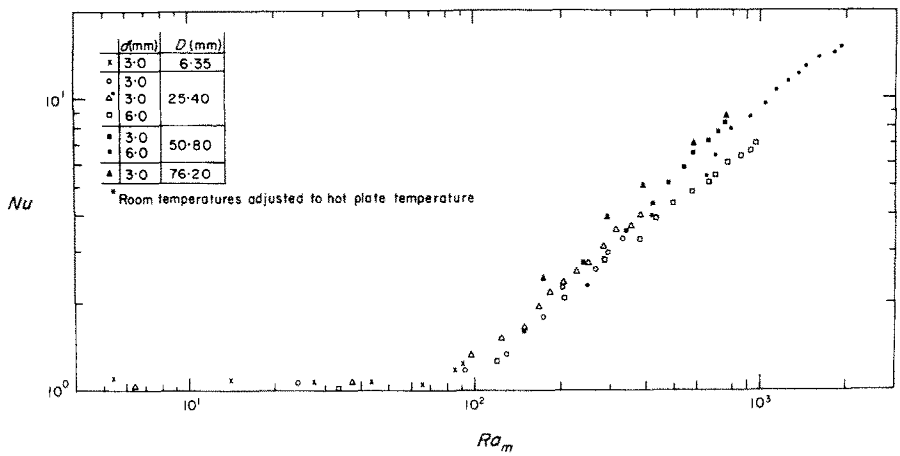

FIG. 2. Relationship between Nu and Ra_m for $T_u = 0$ degC.

[20] can be fairly represented by the work of Gupta and Joseph but with a greater extent of scattering of Combrarnous and LeFur's data. The slope of the heat-transfer curve at $R = R_c = 4\pi^2$ is found by both Masuoka, and Gupta and Joseph to be $1/2\pi^2$. On the other hand, Elder showed that in the vicinity of $R = 4\pi^2$, the relation $Nu \propto R$ should still be satisfactory.

For the two distinctive regions indicated by Masuoka, the following should be stated. In arriving at the expression $Nu = 1+2(1-4\pi^2/R)$, convection is approximated by only the first term. As R increases, the contributions of the second and higher modes to heat transfer become greater. The range of values from $2R_c$ to $3R_c$ is the transition region leading to the higher R region characterized by $Nu \propto R$. As shown by both Elder and Masuoka, heat transfer becomes independent of the layer thickness for higher values of R. Therefore, in these studies there is no evidence that any parameter except R is affecting the heat-transfer process. However, as indicated by Schneider, as R increases and the sharp temperature gradient is confined close to the lower boundary, there may be an influence of the characteristic dimension of the porous medium on heat transfer. Because of this effect, Schneider's results deviate from the linear trend in the high R region shown by Elder and Masuoka. In the region near the onset of convection, the present experimental results coincide with those of Masuoka, Schneider, and Gupta and Joseph. However, for higher values of R, they show a trend closer to the results reported by both Schneider, and Gupta and Joseph, than to those of Elder and Masuoka.

The results of the experiments with $T_u = 0$ degC are shown in Fig. 2. For $Ra_m > 100$, the data can be represented by $Nu = 0{\cdot}024 Ra_m^{0{\cdot}85}$ with a correlation coefficient of 0·984. As predicted by Sun *et al.* [13], when the onset of convection occurs in a porous medium containing a fluid with a density maximum between fixed lower and upper boundary temperatures, the critical modified Rayleigh number is not constant, as in the case of normal fluids (i.e. fluids with monotonic density–temperature relations), but depends on the lower and upper boundary temperatures, and to a slight extent, on the temperature coefficients representing the fluid density–temperature relation. Table 2 shows a comparison between Ra_m and $(Ra_m)_c$ obtained in this investigation. Without exception, for $Ra_m < (Ra_m)_c$, the Nu values are near unity, indicating the conductive mode of heat transfer in the medium. For cases $Ra_m > (Ra_m)_c$, the values of Nu deviate from the value of unity. Three sets of γ_1 and γ_2 were computed for three temperature ranges, i.e. for 0–20 degC, $\gamma_1 = 0{\cdot}793953 \times 10^{-5}$ degC^{-2}, $\gamma_2 = -0{\cdot}655908 \times 10^{-7}$ degC^{-3}; for 0–35 degC, $\gamma_1 = 0{\cdot}776755 \times 10^{-5}$ degC^{-2}, $\gamma_2 = -0{\cdot}522128 \times 10^{-7}$ degC^{-3}; and for 0–60 degC, $\gamma_1 = 0{\cdot}743871 \times 10^{-5}$ degC^{-2}, $\gamma_2 = -0{\cdot}390661 \times 10^{-7}$ degC^{-3}. These three sets of values are used to evaluate Ra_m, and the two thermal parameters λ_1, and λ_2. The $(Ra_m)_c$ values are obtained by a linear interpretation of the results given in Sun *et al.* [13]. As indicated in the table, few experiments fell into either the conductive mode or the transition region.

Figure 3 shows $(Ra_m)_c$ as a function of T_l. It is clear that as T_l increases, the significance of the density maximum effect diminishes, and $(Ra_m)_c$ approaches asymptotically the value of $4\pi^2$; on the other hand, as T_l decreases, $(Ra_m)_c$ increases rapidly. For $T_l = T_m = 4$ degC, the water at the lower boundary will be heavier than the water near the upper boundary. Therefore, the system will remain stable, and the value of Ra_m will approach the limiting value of infinity. In the temperature range covered in this study (6–53·7 degC), $(Ra_m)_c$ is found to be a function of temperature and is no longer a constant value.

Although Nu values can be well correlated by R

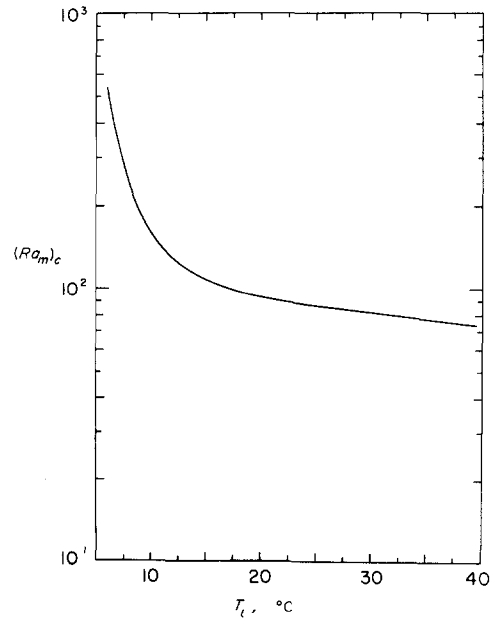

FIG. 3. $(Ra_m)_c$ as a function of T_l.

Table 2. Comparison between Ra_m and $(Ra_m)_c$

D(mm)	d(mm)	T_l(degC)	Nu	λ_1	λ_2	Ra_m	$(Ra_m)_c$
6·35	3·0	6·0	1·07	−2·927	−0·229	0·85	534·1
		11·1	1·10	−1·412	−0·261	5·35	141·3
		16·5	1·08	−1·078	−0·319	14·00	101·0
		22·1	1·06	−0·995	−0·278	26·90	91·9
		27·3	1·06	−0·875	−0·350	43·40	84·8
		33·0	1·10	−0·756	−0·440	66·30	79·4
		36·2	1·17	−0·690	−0·488	84·00	77·3
		39·0	1·24	−0·625	−0·545	94·00	74·3
25·40	3·0	9·1	1·01	−1·664	−0·215	4·58	182·7
		15·3	1·05	−1·134	−0·298	24·15	106·4
		26·5	1·17	−0·895	−0·340	93·60	86·6
		30·9	1·32	−0·811	−0·404	132·00	81·8
25·40	6·0	10·1	1·06	−1·520	−0·224	32·90	156·7
		17·1	1·20	−1·051	−0·330	119·00	99·0
50·80	3·0	25·6	1·45	−0·910	−0·326	133·00	86·5

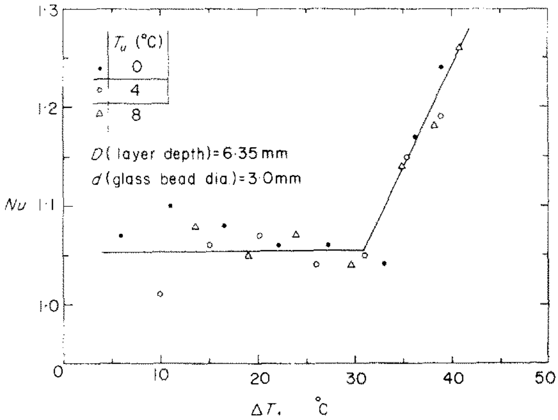

FIG. 4. Relationship between Nu and ΔT for $D = 6{\cdot}35$ mm and $d = 3{\cdot}0$ mm.

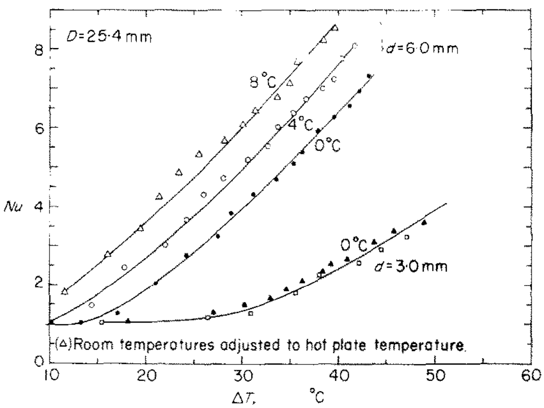

FIG. 5. Relationship between Nu and ΔT for $D = 25{\cdot}4$ mm and $d = 3{\cdot}0, 6{\cdot}0$ mm.

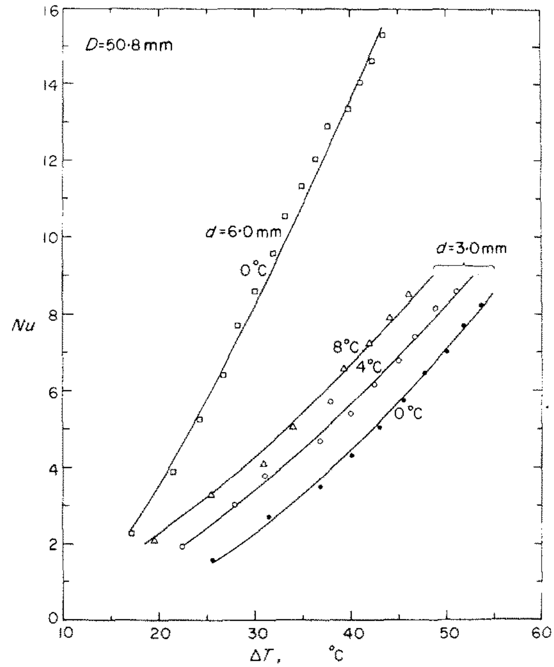

FIG. 6. Relationship between Nu and ΔT for $D = 50{\cdot}8$ mm and $d = 3{\cdot}0, 6{\cdot}0$ mm.

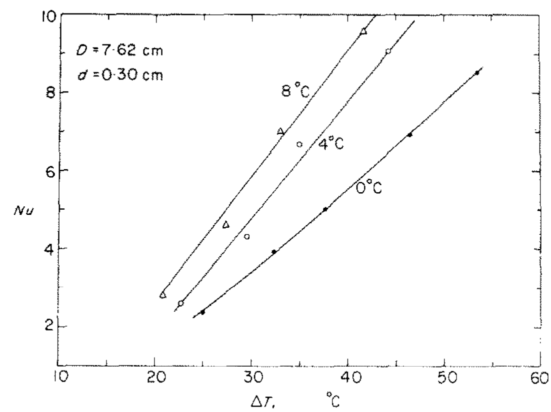

FIG. 7. Relationship between Nu and ΔT for $D = 76{\cdot}2$ mm and $d = 3{\cdot}0$ mm.

and Ra_m as indicated in Figs. 1 and 2, these expressions hardly show the extent and variation of D, and ΔT on the effect of density inversion at 4 degC on heat transfer. To demonstrate this, plots were prepared showing Nu vs ΔT, using T_u as a parameter. For a particular ΔT, the value of T_l will be higher by 4 and 8 degC respectively when T_u is maintained at 4 and 8 degC. For $D = 6{\cdot}35$ mm, $d = 3{\cdot}0$ mm (see Fig. 4), there is no difference in heat-transfer rate for $T_u = 0$, 4 and 8 degC. This is due to the fact that under these experimental conditions, the heat transfer is essentially conductive and is in the transition region. The data for $D = 25{\cdot}4$ mm, $d = 6{\cdot}0$ mm; $D = 50{\cdot}8$ mm, $d = 3{\cdot}0$ mm; and $D = 76{\cdot}2$ mm, $d = 3{\cdot}0$ mm (see Figs. 5–7) exhibit similar trends showing the lowest heat-transfer rate at $T_u = 0$ degC and the significant effect of glass beads diameter on the convective heat-transfer process in the medium.

Table 3 summarizes some ratios of Nu_8/Nu_0 and Nu_4/Nu_0 as functions of ΔT, D and d. It can be noted that the extent of heat transfer is only weakly dependent on ΔT and D. For $D = 25{\cdot}4$ mm, $d = 3{\cdot}0$ mm, the ratios for $\Delta T = 30$ and 40 degC equal unity, since under the experimental conditions, the heat-transfer processes are in either the conductive mode or the transition region. As shown in Figs. 5 and 6, it is clear that for the same D, heat-transfer rates are much higher for $d = 6{\cdot}0$ mm than for $d = 3{\cdot}0$ mm. Since neither the porosities for $d = 6{\cdot}0$ mm and $d = 3{\cdot}0$ mm nor the volume of k_m, varies significantly, it is primarily the pore size that affects the extent of convective motion and facilitates heat transfer.

Table 3. Summary of Nu_8/Nu_0, Nu_4/Nu_0 as functions of ΔT, D, and d

Temperature difference across porous layer, ΔT(degC)	Ratio of Nusselt number at 8 and 4 degC to 0 degC	Layer depth, D(mm)							
		6·35		25·40		50·80		76·20	
		Glass bead, diameter d(mm)							
		3·0	6·0	3·0	6·0	3·0	6·0	3·0	6·0
30	Nu_8/Nu_0	1·00	—	—	1·50	1·90	—	2·00	—
	Nu_4/Nu_0	1·00	—	—	1·20	1·50	—	1·60	—
40	Nu_8/Nu_0	1·00	—	—	1·40	1·50	—	1·60	—
	Nu_4/Nu_0	1·00	—	—	1·20	1·30	—	1·40	—

4. CONCLUSION

1. In this investigation, with $T_u = 4$ and 8 degC (thus eliminating the effect of density inversion on the onset of convection), the critical Rayleigh number was found to be $4\pi^2$, as derived by Lapwood and verified experimentally by Katto and Masuoka. Because of the lack of data in the transition region, the relation

$$Nu = 1 + 2\left[1 - \frac{4\pi^2}{R}\right] \tag{8}$$

developed by Masuoka cannot be verified concretely. In general, the results of the present study are lower than those reported previously. The data for $100 < R < 300$, however, are in excellent agreement with those of a similar study by Masuoka. For higher values of R, the present results are similar to a trend reported by Schneider and to the recent analytical work of Gupta and Joseph.

2. The values of Nu obtained for $T_u = 0$ degC are well correlated with Ra_m by the expression

$$Nu = 0{\cdot}024 Ra_m^{0{\cdot}85} \tag{9}$$

for $Ra_m > 100$, with a correlation coefficient of 0·984. As predicted in an analytical study by Sun *et al.* $(Ra_m)_c$ is found to be dependent on the thermal parameters λ_1 and λ_2 and is no longer a constant value as in the case of normal fluids. As shown in this paper, as T_l increases, the value of $(Ra_m)_c$ decreases and approaches asymptotically a limiting value of $4\pi^2$. When T_l decreases to 4 degC, the temperature possessing the maximum density, the system becomes stable and $(Ra_m)_c$ approaches the limiting value of infinity.

3. The effect of 4 degC on convective heat transfer is significant and was found to be dependent on ΔT. For small ΔT, the value of Nu for $T_u = 0$ degC can be lower by 100 per cent than for $T_u = 4$ and 8 degC. The effect of the filling particle diameter was also found to be significant. This is due to the pore size and its effect on the extent of convective motion.

Acknowledgement—Sponsorship of this research by the Cold Regions Research and Engineering Laboratory, U.S. Army Corps of Engineers, is greatly appreciated. The author wishes to express his thanks to Dr. Chi Tien of Syracuse University for his helpful discussions during the preparation of this manuscript.

REFERENCES

1. C. W. Horton and F. T. Rogers, Jr., Convection currents in a porous medium, *J. Appl. Phys.* **16**, 367 (1945).
2. H. L. Morrison, F. T. Rogers, Jr. and C. W. Horton, Convection currents in porous media, II. Observation of conditions at onset of convection, *J. Appl. Phys.* **20**, 1027 (1949).
3. F. T. Rogers, Jr. and H. L. Morrison, Convection currents in porous media, III. Extended theory of the critical gradient, *J. Appl. Phys.* **21**, 1177 (1950).
4. F. T. Rogers, Jr., L. E. Schilberg and H. L. Morrison, Convection currents in porous media, IV. Remarks on the theory, *J. Appl. Phys.* **22**, 1476 (1951).
5. E. R. Lapwood, Convection of a fluid in a porous medium, *Proc. Camb. Phil. Soc.* **44**, 508 (1948).
6. Y. Katto and T. Masuoka, Criterion for the onset of convection flow in a fluid in a porous medium, *Int. J. Heat Mass Transfer* **10**, 297 (1967).
7. J. W. Elder, Steady free convection in a porous medium heated from below, *J. Fluid Mech.* **27**, 29 (1967).
8. P. S. Karra, M.Sc. Thesis, University of Calgary, Alberta, Canada (1968).
9. W. R. Debler, On the analogy between thermal and rotational hydrodynamic stability, *J. Fluid Mech.* **24**, 165 (1966).
10. G. Vernois, Penetrative convection, *Astrophys. Jl* **137**, 641 (1963).
11. C. Tien, Thermal instability of a horizontal layer of water near 4 degC, *A.I.Ch.E. Jl* **6**, 71 (1960).
12. Z. S. Sun, C. Tien and Y. C. Yen, Thermal instability of a horizontal layer of liquid with maximum density, *A.I.Ch.E. Jl* **15**, 910 (1969).
13. Z. S. Sun, C. Tien and Y. C. Yen, Onset of convection in a porous medium containing liquid with a density maximum, *Proceedings of Fourth International Heat Transfer Conference (Paris–Versailles)*, Vol. IV, NC2-11 (1972).
14. T. Masuoka, Heat transfer by free convection in a porous layer heated from below, *Heat Transfer—Japan. Res.* **1**, 39 (1972).

15. C. Tien, Y. C. Yen and J. W. Dotson, Free convective heat transfer in a horizontal layer of liquid—the effect of density inversion, *A.I.Ch.E. Sym. Ser.* **68**(118), 101 (1972).
16. D. Kunii and J. M. Smith, Heat transfer characteristics of porous rocks, *A.I.Ch.E. Jl* **6**, 71 (1960).
17. K. J. Schneider, Investigation of the influence of free convection on heat transfer through granular material, *11th Int. Cong. of Refrigeration* (*Munich*), paper, 11-4 (1963).
18. V. P. Gupta and D. D. Joseph, Bounds for heat transport in a porous layer, *J. Fluid Mech.* **57**, 491 (1973).
19. M. Combrarnous and B. LeFur, Transfert de chaleur par convection naturelle dans une couche pureuse horizontale, *C.R. Hebd. Séanc. Acad. Sci., Paris* **269**, 1009 (1969).
20. R. Buretta and A. S. Berman, Convective heat transfer in a liquid-saturated porous layer. To be published.

EFFET DE L'INVERSION DE DENSITE SUR LE TRANSFERT THERMIQUE PAR CONVECTION NATURELLE DANS UNE COUCHE POREUSE CHAUFFEE PAR LE BAS

Résumé—On analyse l'effet de l'inversion de densité sur le transfert thermique par convection libre dans une couche poreuse chauffée par le bas. Des billes de verre constituent le milieu poreux. Pour la frontière supérieure à 4 et 8°C, ce qui élimine l'effet de l'inversion de densité, le nombre de Rayleigh critique est égal à $4\pi^2$. L'influence de l'inversion de densité est évaluée en maintenant la frontière supérieure à 0°C. L'établissement de la convection dépend de deux paramètres thermiques qui sont fonctions des températures aux limites et des coefficients qui représentent la dépendance de la masse volumique vis à vis de la température. Le nombre de Nusselt peut être représenté en fonction du nombre de Rayleigh modifié. L'effet de l'inversion de densité sur le transfert thermique est très sensible et il décroit lorsque la différence de température à travers la couche augmente. Pour des faibles ΔT, l'effet de l'inversion de densité est tel que le transfert thermique peut être moitié de celui qui correspond à l'absence d'inversion de densité.

EINFLUSS VON DICHTE-INVERSION AUF DEN WÄRMEÜBERGANG BEI FREIER KONVEKTION IN EINER PORÖSEN, VON UNTEN BEHEIZTEN SCHICHT

Zusammenfassung—Es wurde eine Studie durchgeführt, um die Einflüsse einer Dichte-Inversion auf den Wärmeübergang bei freier Konvektion in einer porösen, von unten beheizten Schicht zu untersuchen. Glaskörner in Wasser stellten das poröse Medium dar. Für die Temperaturen von 4 und 8°C an der Oberseite, bei denen der Einfluß von Dichte-Inversion auf das Einsetzen der Konvektion ausgeschlossen war, ergab sich die kritische Rayleigh-Zahl zu $4\pi^2$.

Der Einfluß der Dichte-Inversion wurde durch Aufrechterhalten von 0°C an der Oberseite zur Geltung gebracht. Es ergab sich, daß das Einsetzen der Konvektion von zwei thermischen Parametern abhängt, die Funktionen der Grenztemperaturen und der Koeffizienten sind, durch welche die Temperaturabhängigkeit der Dichte ausgedrückt wird.

Die Nusselt-Zahl kann als Funktion einer modifizierten Rayleigh-Zahl dargestellt werden. Es ergab sich ein signifikanter Einfluß der Dichte-Inversion auf die übertragene Wärme, der mit steigender Temperaturdifferenz in der Schicht abnimmt. Für kleine ΔT führt die Dichte-Inversion zum Rückgang der übertragenen Wärme um 100% gegenüber Bedingungen ohne Dichte-Inversion.

ВЛИЯНИЕ ИНВЕРСИИ ПЛОТНОСТИ НА ТЕПЛООБМЕН ПРИ СВОБОДНОЙ КОНВЕКЦИИ В НАГРЕВАЕМОМ СНИЗУ ПОРИСТОМ СЛОЕ

Аннотация — Исследуется влияние инверсии плотности на теплообмен при свободной конвекции в нагреваемом снизу пористом слое, состоящем из стекляных шариков и воды. Найдено, что критическое число Релея равно $4\pi^2$ при температуре верхней границы 4° и 8°С, когда исключается влияние инверсии плотности на возникновение конвекции. Влияние инверсии плотности определялось при температуре верхней границы, равной °С. Найдено, что возникновение конвекции зависит от двух тепловых параметров, которые являются функциями температур на границах слоя и коэффициентов, представляющих зависимость плотности жидкости от температуры. Число Нуссельта можно представить в виде модифицированного числа Релея. Найдено, что влияние инверсии плотности на интенсивность теплообмена является существенным и уменьшается с увеличением разности температур через слой. При малом значении ΔT инверсия плотности уменьшает интенсивность теплообмена на 100% по сравнению со случаем отсутствия инверсии плотности.